

# Isovector and isoscalar pairing in odd-odd $N = Z$ nuclei within a quartet approach

D. Negrea and N. Sandulescu

*National Institute of Physics and Nuclear Engineering,  
76900 Bucharest-Magurele, Romania*

D. Gambacurta

*Extreme Light Infrastructure - Nuclear Physics (ELI-NP),  
76900 Bucharest-Magurele, Romania*

## Abstract

The quartet condensation model (QCM) is extended for the treatment of isovector and isoscalar pairing in odd-odd  $N=Z$  nuclei. In the extended QCM approach the lowest states of isospin  $T=1$  and  $T=0$  in odd-odd nuclei are described variationally by trial functions composed by a proton-neutron pair appended to a condensate of 4-body operators. The latter are taken as a linear superposition of an isovector quartet, built by two isovector pairs coupled to the total isospin  $T=0$ , and two collective isoscalar pairs. In all pairs the nucleons are distributed in time-reversed single-particle states of axial symmetry. The accuracy of the trial functions is tested for realistic pairing Hamiltonians and odd-odd  $N=Z$  nuclei with the valence nucleons moving above the cores  $^{16}\text{O}$ ,  $^{40}\text{Ca}$  and  $^{100}\text{Sn}$ . It is shown that the extended QCM approach is able to predict with high accuracy the energies of the lowest  $T=0$  and  $T=1$  states. The present calculations indicate that in these states the isovector and the isoscalar pairing correlations coexist together, with the former playing a dominant role.

## I. INTRODUCTION

Many experimental and theoretical studies have been dedicated lately to the role played by the isoscalar and isovector proton-neutron (pn) pairing in odd-odd  $N=Z$  nuclei (e.g., see [1, 2] and the references quoted therein). The experimental data show that the ground states of odd-odd  $N=Z$  nuclei have the isospin  $T=0$  for  $A < 34$  and, with some exceptions, the isospin  $T=1$  for heavier nuclei. This fact is sometimes considered as an indication of the dominant role of isoscalar ( $T=0$ ) pn pairing in light  $N=Z$  nuclei. The fingerprints of  $T=0$  pn pairing in odd-odd  $N=Z$  nuclei is also investigated lately in relation to the Gamow-Teller (GT) charge-exchange reactions. Thus in some odd-odd  $N=Z$  nuclei there is an enhancement of the GT strength in the low-energy region which appears to be sensitive to the  $T=0$  pn interaction [3]. The competition between the isovector and isoscalar pairing in odd-odd nuclei was also discussed extensively in relation to the odd-even mass difference along  $N=Z$  line [4, 5].

On theoretical side, the role of pn pairing in odd-odd  $N=Z$  nuclei is still not clear. A fair description of low-lying states and GT transitions in odd-odd  $N=Z$  nuclei is given by the shell model (SM) calculations (e.g., see [6]). However, due to the complicated structure of the SM wave function, from these calculations it is not easy to draw conclusions on the role played by the pn pairing. Recently, the effect of  $T=0$  and  $T=1$  pairing forces on the spectroscopic properties of odd-odd  $N=Z$  nuclei was analyzed in the framework of a simple three-body model in which the odd pn pair is supposed to move on the top of a closed even-even core [7]. This model gives good results for the nuclei in which the core can be considered as inert, such as  $^{18}\text{F}$  and  $^{42}\text{Sc}$ , but not for the nuclei in which the core degrees of freedom are important.

The difficulties mentioned above point to the need of new microscopic models which, on one hand, to be able to describe reasonably well the spectroscopic properties of odd-odd  $N=Z$  nuclei, and, on the other hand, to be simple enough for understanding the impact of pn pairing correlations on physical observables. As an alternative, in this article we shall use the framework of the quartet condensation model (QCM) we have proposed in Ref. [8]. Its advantage is the explicit treatment of the pairing correlations in the wave function and, compared to other pairing models, the exact conservation of particle number and isospin. The scope of this study is to extend the QCM approach of Ref. [8], applied previously to

even-even nuclei, for the case of odd-odd N=Z nuclei and to study, for these nuclei, the role played by proton-neutron pairing in the lowest T=0 and T=1 states.

## II. FORMALISM

In the present study the isovector and isoscalar pairing correlations in odd-odd N=Z nuclei are described by pairing forces which act on pairs of nucleons moving in time-reversed single-particle states generated by axially-deformed mean fields. The corresponding Hamiltonian is given by

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P_{i,t}^+ P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D_{i,0}^+ D_{j,0}. \quad (1)$$

In the first term  $\varepsilon_{i\tau}$  are the single-particle energies for the neutrons ( $\tau = 1/2$ ) and protons ( $\tau = -1/2$ ) while  $N_{i\tau}$  are the particle number operators. The second term is the isovector pairing interaction expressed by the pair operators  $P_{i,0}^+ = (\nu_i^+ \pi_{\bar{i}}^+ + \pi_i^+ \nu_{\bar{i}}^+)/\sqrt{2}$ ,  $P_{i,1}^+ = \nu_i^+ \nu_{\bar{i}}^+$  and  $P_{i,-1}^+ = \pi_i^+ \pi_{\bar{i}}^+$ , where  $\nu_i^+$  and  $\pi_i^+$  are creation operators for neutrons and protons in the state  $i$ . The last term is the isoscalar pairing interaction represented by the operators  $D_{i,0}^+ = (\nu_i^+ \pi_{\bar{i}}^+ - \pi_i^+ \nu_{\bar{i}}^+)/\sqrt{2}$  which creates a non-collective isoscalar pair in the time reversed states  $(i, \bar{i})$ . In the applications considered in the present paper the single-particle states have axial symmetry.

The Hamiltonian (1) was employed recently to study the isovector and isoscalar pairing correlations in even-even N=Z nuclei in the framework of QCM approach [8]. This approach is extended here for the case of odd-odd nuclei. For consistency reason we start by presenting shortly the QCM approach for even-even nuclei.

In the QCM approach the ground state of the Hamiltonian (1) for a system of N neutrons and Z protons, with N=Z=even, moving above a closed core  $|0\rangle$  is described by the ansatz

$$|QCM\rangle = (A^+ + (\Delta_0^+)^2)^{n_q} |0\rangle, \quad (2)$$

where  $n_q = (N + Z)/4$ . The operator  $A^+$  is the collective quartet defined by a superposition of two non-collective isovector pairs coupled to total isospin T=0 and has the expression

$$A^+ = \sum_{i,j} x_{ij} [P_i^+ P_j^+]^{T=0}. \quad (3)$$

Supposing that the mixing amplitudes  $x_{ij}$  are separable, that is  $x_{ij} = x_i x_j$ , the collective quartet gets the form

$$A^+ = 2\Gamma_1^+ \Gamma_{-1}^+ - (\Gamma_0^+)^2, \quad (4)$$

where  $\Gamma_t^+ = \sum_i x_i P_{i,t}^+$  are the collective neutron-neutron ( $t=1$ ), proton-proton ( $t=-1$ ) and proton-neutron ( $t=0$ ) isovector pairs. Finally, in Eq. (2) the operator  $\Delta_0^+$  is the collective isoscalar pair defined by

$$\Delta_0^+ = \sum_i y_i D_{i,0}^+. \quad (5)$$

When the single-particle states are degenerate and the strength of the two pairing forces are equal, the QCM state (2) is the exact solution of the Hamiltonian (1). For realistic single-particle spectra and realistic pairing interactions the QCM state (2) is not anymore the exact solution but, as shown in Ref. [8], it predicts with high accuracy the pairing correlations in even-even  $N=Z$  nuclei.

In what follows we extend the QCM approach to odd-odd  $N=Z$  systems. The main assumption, suggested by the exact solution of the Hamiltonian (1) (see below), is that the lowest  $T=1$  and  $T=0$  states in odd-odd nuclei can be well described variationally by trial states obtained by appending to the QCM function (2) a proton-neutron pair. Since the isospin of the QCM state (2) is  $T=0$ , the total isospin of the odd-odd system is given by the isospin of the appended pair. Thus, the ansatz for the lowest  $T=1$  state of the odd-odd  $N=Z$  systems is

$$|iv; QCM\rangle = \tilde{\Gamma}_0^+ (A^+ + (\Delta_0^+)^2)^{n_q} |0\rangle, \quad (6)$$

where  $\tilde{\Gamma}_0^+ = \sum_i z_i P_{i,0}^+$  is the isovector pn pair attached to the the even-even part of the state (in what follows we shall use the name "core" for the even-even part of the state (6), which should be not confused with the closed core  $|0\rangle$ ). It can be seen that this pair has a different collectivity compared to the isovector pn pair  $\Gamma_0^+$  contained in the quartet  $A^+$  (see Eq. 4).

Likewise, the lowest  $T=0$  state of odd-odd  $N=Z$  systems is described by the function

$$|is; QCM\rangle = \tilde{\Delta}_0^+ (A^+ + (\Delta_0^+)^2)^{n_q} |0\rangle, \quad (7)$$

where  $\tilde{\Delta}_0^+ = \sum_i z_i D_{i,0}^+$  is the odd isoscalar pair, which has also a different structure compared to the isoscalar pair  $\Delta_0^+$  which enters in the even-even core. Due to its different isospin, the state (7) is orthogonal to the isovector state (6).

We have proved that the states (6,7) are the exact eigenfunctions of the Hamiltonian (1) when the single-particle energies are degenerate and when the pairing forces have the same strength, i.e.,  $V^{T=1}(i, j) = V^{T=0}(i, j) = g$ . In this case the states (6,7) have the same energy which, for  $\epsilon_i = 0$ , is given by

$$E(n_q, \nu) = (\nu - 2n_q)g + 2n_q(\nu - n_q + 2)g, \quad (8)$$

where  $n_q$  is the number of quartets and  $\nu$  is the number of single-particle levels. In Eq. (8) the second term corresponds to the energy of the even-even core of the functions (6,7). It is worth to be mentioned that this exact solution is not the exact solution of the SU(4) model [9] because in the Hamiltonian (1) the isoscalar force contains only pairs in time-reversed single-particle states.

For a non-degenerate single-particle spectrum and general pairing forces the QCM states (6,7) are determined variationally. The variational parameters are the amplitudes  $x_i$ ,  $y_i$  and  $z_i$  which are defining, respectively, the isovector pairs  $\Gamma_t^+$ , the isoscalar pair  $\Delta_0^+$  and the odd pn pair. They are found by minimizing the average of Hamiltonian (1) on the QCM states (6,7) and by imposing, for the latter, the normalization condition. The average of the Hamiltonian and the norm of the QCM states are calculated using the technique of recurrence relations. More precisely, the calculations are performed using auxiliary states composed by products of collective pairs. Thus, for the isovector T=1 state (6) the auxiliary states are

$$|n_1 n_2 n_3 n_4 n_5\rangle = (\Gamma_1^+)^{n_1} (\Gamma_{-1}^+)^{n_2} (\Gamma_0^+)^{n_3} (\Delta_0^+)^{n_4} (\tilde{\Gamma}_0^+)^{n_5} |0\rangle. \quad (9)$$

The auxiliary states for the calculations of isoscalar T=0 state (7) have a similar structure with the difference that the odd isovector pair  $\tilde{\Gamma}_0^+$  is replaced by the odd isoscalar pair  $\tilde{\Delta}_0^+$ . It can be observed that the QCM states (6,7) can be expressed in terms of a subset of auxiliary states corresponding to specific combinations of  $n_i$ . However, in order to close the recurrence relations one needs to evaluate the matrix elements of the Hamiltonian (1) for all auxiliary states which satisfy the conditions  $\sum_i n_i = (N + Z)/2$  and  $n_5 = 0, 1$ . An example of recurrence relations, for the case of even-even systems, can be seen in Refs. [10, 11].

The advantage of the QCM approach is the possibility to investigate in a direct manner the role of various types of correlations by simply switching them on and off in the structure of the states (6,7). Thus, in order to explore the importance of isoscalar pairing on the lowest T=0 and T=1 states in odd-odd N=Z systems one can remove from the functions

(6,7) the isoscalar pair  $\Delta_0^+$ . In this approximation the functions get the expressions

$$|is; Q_{iv}\rangle = \tilde{\Delta}_0^+(A^+)^{n_q}|0\rangle, \quad (10)$$

$$|iv; Q_{iv}\rangle = \tilde{\Gamma}_0^+(A^+)^{n_q}|0\rangle. \quad (11)$$

Alternatively, we can estimate the importance of the isovector pairing by removing from the QCM functions the isovector quartet  $A^+$ . The corresponding functions are

$$|C_{is}\rangle = (\Delta_0^+)^{2n_q+1}|0\rangle, \quad (12)$$

$$|iv; C_{is}\rangle = \tilde{\Gamma}_0^+(\Delta_0^{+2})^{n_q}|0\rangle. \quad (13)$$

Another possibility is to remove from the QCM functions the contribution of like-particle pairs, keeping only the isovector and isoscalar pn pairs. These trial states, which can be employed to study the role of like-particle pairing in N=Z nuclei, have the expressions

$$|is; C_{iv}\rangle = \tilde{\Delta}_0^+(\Gamma_0^{+2})^{n_q}|0\rangle, \quad (14)$$

$$|C_{iv}\rangle = (\Gamma_0^+)^{2n_q+1}|0\rangle. \quad (15)$$

Contrary to the previous approximations, the states (14,15) have not a well-defined isospin.

Among the approximations mentioned above of special interest are the ones corresponding to the states (12) and (15), which are pure condensates of isoscalar and, respectively, isovector pn pairs. These states are sometimes considered as representative for understanding the competition between isovector and isoscalar proton-neutron pairing in nuclei.

The QCM states (6,7) and all the approximations based on them are formulated here in the intrinsic system associated to the axially deformed single-particle levels. Therefore they have a well-defined projection of the angular momentum on z-axis but not a well-defined angular momentum. A more complicated quartet formalism for odd-odd nuclei, which conserves exactly the angular momentum and takes into account the correlations induced by a general two-body force, was proposed recently in Ref. [12].

### III. RESULTS

To test the accuracy of the QCM approach for odd-odd N=Z nuclei we consider nuclei having protons and neutrons outside the closed cores  $^{16}\text{O}$ ,  $^{40}\text{Ca}$  and  $^{100}\text{Sn}$ . The calculations

are performed employing for the pairing forces and the single-particle states a similar input as in our previous study for even-even nuclei [8]. Thus, the single-particle states are generated by axially deformed mean fields calculated with the Skyrme-HF code *ev8* [13] and with the force *Sly4* [14]. In the mean field calculations the Coulomb interaction is switched off, so the single-particle energies for protons and neutrons are the same. For the pairing forces we use a zero range delta interaction  $V^T(r_1, r_2) = V_0^T \delta(r_1 - r_2) \hat{P}_{S, S_z}^T$ , where  $\hat{P}_{S, S_z}^T$  is the projection operator on the spin of the pairs, i.e.,  $S = 0$  for the isovector ( $T=1$ ) force and  $S = 1, S_z = 0$  for the isoscalar ( $T=0$ ) force. The matrix elements of the pairing forces are calculated using the single-particle wave functions generated by the Skyrme-HF calculations (for details, see [15]). As parameters we use the strength of the isovector force, denoted by  $V_0$ , and the scaling factor  $w$  which defines the strength of the isoscalar force,  $V_0^{T=0} = wV_0$ . How to fix these parameters is not a simple task. Since the main goal of this study is to test the accuracy of the QCM approach, we have made several calculations with various strengths,  $V_0 = \{300, 465, 720, 1000\}$  MeV fm $^{-3}$ , which cover all possible situations, from the weak to the strong pairing regime. Because the conclusions relevant for this study are quite similar for all these strengths, here we are presenting only the results for the pairing strength  $V_0 = 465$  MeV fm $^{-3}$  employed in our previous study of even-even nuclei [8].

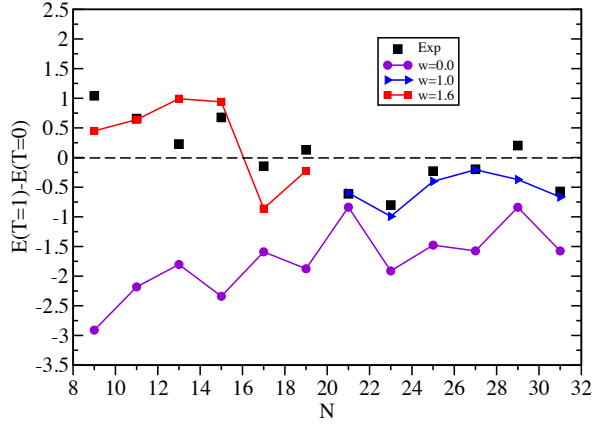


FIG. 1: The energy difference between the lowest  $T=1$  and  $T=0$  states as a function of  $N=Z=A/2$ . The experimental data are extracted from Ref. [16]. The solid lines show the exact results obtained by diagonalising the Hamiltonian (1). The calculations correspond to the strength  $V_0=465$  MeV fm $^{-3}$  and to various scaling factors  $w$ .

For the scaling factor  $w$  we also used various values,  $w = \{1.0, 1.3, 1.5, 1.6\}$ . To find the most appropriate value of  $w$  for the strength  $V_0 = 465 \text{ MeV fm}^{-3}$  we have searched for the best agreement with the energy difference between the first excited state and the ground state of odd-odd nuclei. These energy differences are shown in Fig. 1 by black squares. It is worth mentioning that the lowest  $T=0$  state can have various angular momenta  $J \geq 1$  (e.g., the ground states of  $^{22}\text{Na}$  and  $^{26}\text{Al}$  have  $J = 3$  and, respectively,  $J = 5$ ). The theoretical results shown in Fig. 1 corresponds to the exact diagonalization of Hamiltonian (1) in a space spanned by 10 single-particle levels above the cores  $^{16}\text{O}$  and  $^{40}\text{Ca}$ . The best agreement with the experimental data is obtained by choosing  $w = 1.6$  for  $sd$ -shell nuclei and  $w = 1.0$  for  $pf$ -shell nuclei. As seen in Fig. 1, for these parameters the calculations predict rather well how the isospin of the ground state is changing with the mass number. Since for the nuclei above  $^{100}\text{Sn}$  there are no available experimental data on low-lying states which to be used for fixing the scaling factor  $w$ , in the calculations presented below we have chosen for  $w$  the same value as for the  $pf$ -shell nuclei. In Fig. 1 we show also the results obtained considering only the isovector pairing force, that is, for  $w = 0.0$ . It can be seen that in this case the predictions are quite far from the data, especially for the  $sd$ -shell nuclei.

With the parameters of the Hamiltonian fixed as explained above, we have studied how accurate are the energies of the lowest  $T=0$  and  $T=1$  states predicted by the extended QCM approach for the odd-odd nuclei. The results are presented in Table I. Are shown the correlation energies defined as  $E_{corr} = E_0 - E$ , where  $E$  is the total energy while  $E_0$  is the non-interacting energy obtained by switching off the pairing interactions. The correlation energies predicted by the QCM functions (6,7) are given in the 4th column. In the brackets are indicated the errors relative to the exact energies shown in the 3rd column. It can be observed that for all the states and nuclei shown in Table I the errors are small, under 1%. We can thus conclude that the QCM functions (6,7) provide an accurate description of the lowest  $T=0$  and  $T=1$  states of the Hamiltonian (1).

One of the advantages of the QCM approach is the opportunity to study the relevance of various types of pairing correlations directly through the structure of the trial states (6,7). As discussed in the previous Section, this is possible by using the approximations (10-15). The correlation energies corresponding to these approximations are shown in Table I. In brackets are given the errors relative to the exact results. One can observe that the smallest errors correspond to the approximations (10,11) in which the contribution of the



TABLE I: Correlation energies, in MeV, for the lowest T=1 and T=0 states. In the brackets are given the errors relative to the exact values indicated in the 3rd column. Are shown the results corresponding to the QCM states (6,7) and to the approximations defined by Eqs. (10-15).

		Exact	$ QCM\rangle$	$ iv; QCM_{iv}\rangle/ is; QCM_{iv}\rangle$	$ iv; C_{is}\rangle/ C_{is}\rangle$	$ C_{iv}\rangle/ is; C_{iv}\rangle$
<sup>22</sup> Na	T=0	13.87	13.87 (0.00%)	13.86 (0.07%)	13.85 (0.12%)	13.85 (0.15%)
	T=1	13.23	13.23 (0.03%)	13.22 (0.05%)	12.97 (1.97%)	13.22 (0.11%)
<sup>26</sup> Al	T=0	22.06	22.05 (0.03%)	22.04 (0.07%)	21.94 (0.53%)	21.79 (1.24%)
	T=1	21.07	21.06 (0.02%)	21.05 (0.07%)	20.93 (0.66%)	20.98 (0.41%)
<sup>30</sup> P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)
	T=1	11.72	11.66 (0.44%)	11.62 (0.82%)	10.94 (7.11%)	10.96 (6.94%)
<sup>46</sup> V	T=1	7.92	7.92 (0.04%)	7.91 (0.10%)	7.33 (8.11%)	7.76 (2.11%)
	T=0	6.93	6.93 (0.01%)	6.93 (0.07%)	6.73 (2.99%)	6.79 (2.05%)
<sup>50</sup> Mn	T=1	12.77	12.76 (0.07%)	12.75 (0.14%)	12.52 (2.02%)	12.62 (1.22%)
	T=0	12.37	12.36 (0.04%)	12.34 (0.24%)	12.18 (1.61%)	12.19 (1.48%)
<sup>54</sup> Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)
	T=0	15.93	15.92 (0.04%)	15.89 (0.22%)	15.53 (2.56%)	15.66 (1.73%)
<sup>106</sup> I	T=1	5.15	5.14 (0.08%)	5.13 (0.23%)	4.71 (9.37%)	4.93 (4.51%)
	T=0	4.53	4.52 (0.04%)	4.51 (0.42%)	4.19 (7.84%)	4.29 (5.53%)
<sup>110</sup> Cs	T=1	8.03	7.98 (0.56%)	7.97 (0.75%)	7.16 (12.14%)	7.59 (5.86%)
	T=0	7.09	7.06 (0.45%)	7.04 (0.80%)	6.47 (9.64%)	6.65 (6.77%)
<sup>114</sup> La	T=1	9.76	9.72 (0.36%)	9.69 (0.73%)	8.79 (11.03%)	9.27 (5.23%)
	T=0	8.95	8.93 (0.28%)	8.92 (0.42%)	8.31 (7.74%)	8.51 (5.18%)

isoscalar pairs in the even-even core of the QCM functions is neglected. It can be seen that, compared to the calculations with the full QCM functions, in these approximations the errors are increasing by 2-3 times for T=1 states and by larger factors for some T=0 states. However, all the errors relative to the exact results remain under 1%.

In column 6 are shown the results corresponding to the approximations (12,13) in which the isovector quartet is taken out from the even-even core. We can see that in this case the

errors are much bigger than in the case when the isoscalar pairs are neglected. In the last column are given the results of approximations (14,15) obtained by neglecting in the QCM states the contribution of the like-particle pairs. It can be noticed that for all nuclei the states  $T=1$  are better described by a condensate of isovector pn pairs rather than by the approximation (13). On the other hand, the ground  $T=0$  states of  $sd$ -shell nuclei are slightly better described by a condensate of isoscalar pn pairs rather than the approximation (14). However, the latter approximation is by far better than the former in the case of excited  $T=0$  states of  $pf$ -shell nuclei and nuclei with  $A > 100$ .

Overall, these calculations show that the  $T=0$  and  $T=1$  states cannot be well described as pure condensates of isoscalar and, respectively, isovector pairs. In general, by neglecting the contribution of like-particle pairs are generated large errors. The best approximation, for both  $T=0$  and  $T=1$  states, is the one in which the odd pn pair is appended to a condensate of isovector quartets. This fact indicates that the 4-body quartet correlations play an important role in odd-odd  $N=Z$  nuclei. As demonstrated in [10], these correlations are missed when the condensate of isovector quartets is replaced by products of pair condensates.

For understanding better how the different pairing modes are contributing to the total energy, in Fig.2 are shown the isovector and isoscalar pairing energies for the ground states of  $sd$  and  $pf$  nuclei. The pairing energies are calculated by averaging the corresponding pairing forces on the QCM functions (6,7). It is important to be observed that the pairing energies for  $T=1$  ( $T=0$ ) states include also contributions from the isoscalar (isovector) pairing correlations, a fact which is coming from the mixing of isovector and isoscalar degrees of freedom through the even-even core of the QCM functions.

In the left panel of Fig. 2 are plotted the pairing energies in the ground  $T=0$  states of  $sd$ -shell nuclei. As a reference is shown the pairing energy  $E_{pn}^{T=0}$  for  $^{18}\text{F}$ , which corresponds to one  $T=0$  pair above  $^{16}\text{O}$ . It can be seen that the curves for  $E_{pn}^{T=0}$  and  $E_{pn}^{T=1}$  are almost parallel. This indicates that the extra pairing energy in the  $T=0$  channel for  $A > 18$  is related mainly to the contribution of the odd pn  $T=0$  pairs. It is also worth noticing that the total pairing energy in the  $T=1$  channel contains also the contribution from the proton-proton (pp) and neutron-neutron (nn) pairing energies, which, due to the isospin symmetry, are equal to the pn  $T=1$  pairing energy. Therefore, the total isovector pairing energy is comparable to the isoscalar pairing energy, although the latter contains in addition a large contribution from the extra odd  $T=0$  pair.

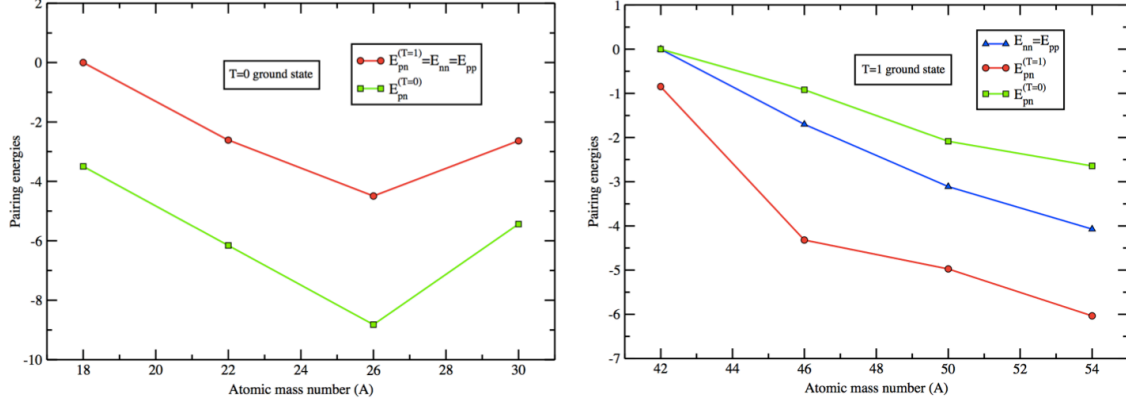


FIG. 2: Pairing energies, in MeV, for the odd-odd  $N=Z$  nuclei as a function of the mass number  $A$ . In the left (right) panel are shown the results for the  $sd$ -shell ( $pf$ -shell) nuclei.

In the right panel of Fig. 2 are plotted the pairing energies for the  $T=1$  ground states of  $pf$ -shell nuclei. It can be seen that  $E_{pn}^{T=0}$  is smaller than  $E_{pn}^{T=1}$  and also smaller than the like-particle pairing energy. At variance with what seen in the left panel, the energy difference  $E_{pn}^{T=1} - E_{pn}^{T=0}$  for  $A > 42$  is much larger than the energy of the odd pn  $T=1$  pair in  $^{42}\text{Sc}$ . Therefore, the larger pn pairing energy in the isovector channel cannot be related only to the extra pn  $T=1$  pair attached to the even-even core. This fact can be traced back to the strong increase of  $E_{pn}^{T=1}$  from  $A=42$  to  $A=46$ . This increase is mainly related to the contribution, in the nucleus  $A=46$ , of the two pn  $T=1$  pairs from the isovector quartet. Since in the isovector quartet all  $T=1$  pairs have the same structure, the pairing energy of these pn  $T=1$  pairs are equal to the pairing energies of like-particle pairs, which, as seen in  $A=46$ , are large, even larger than the energy of the odd pair.

The  $T=0$  states in odd-odd  $N=Z$  nuclei are often described as states having a two quasi-particle structure. Thus, to evaluate the energies of  $T=0$  states it is commonly employed the blocking procedure, which means that the odd  $T=0$  pair is not considered as a collective

TABLE II: Schmidt numbers for the proton-neutron pairs in the lowest T=1 and T=0 states of various odd-odd N=Z nuclei. By  $K_x$  and  $K_y$  are denoted the Schmidt numbers for the pairs  $\Gamma_0^+$  and  $\Delta_0^+$  while  $K_z$  is the Schmidt number for the odd pair, i.e.,  $\tilde{\Gamma}_0^+$  for T=1 states and  $\tilde{\Delta}_0^+$  for T=0 states.

	<sup>26</sup> Al		<sup>30</sup> P		<sup>50</sup> Mn		<sup>54</sup> Co		<sup>110</sup> Cs		<sup>114</sup> La	
	T=1	T=0	T=1	T=0	T=1	T=0	T=1	T=0	T=1	T=0	T=1	T=0
$K_x$	1.25	1.92	3.05	3.05	1.47	1.41	2.37	2.36	1.64	1.66	3.18	3.09
$K_y$	1.97	1.31	1.89	1.56	2.39	1.33	1.72	1.25	2.24	1.88	1.16	1.24
$K_z$	2.77	1.63	2.82	1.65	1.99	1.09	2.30	1.63	2.34	1.29	4.09	1.33

pair in which the nucleons are scattered on nearby single particle levels but just as a proton and a neutron sitting on a single level. In what follows we are going to examine the validity of this approximation in the framework of the QCM approach. In order to analyze this issue, we need an working definition for the collectivity of a pair. Here we shall use the so-called Schmidt number, which is commonly employed to analyze the entanglement of composite systems formed by two parts [17]. In the case of a pair operator  $\Gamma^+ = \sum_{i=1}^{n_s} w_i a_i^+ a_i^+$  the Schmidt number has the expression  $K = (\sum_i \omega_i^2)^2 / \sum_i \omega_i^4$  (for an application of K to like-particle pairing see Ref. [18]). When there is no entanglement  $K=1$  while when the entanglement is maximum, which means equal occupancy of all available states,  $K = n_s$ , where  $n_s$  is the number of states. As examples, in Table II we show for some nuclei the Schmidt numbers corresponding to the pairs which compose the QCM states (6,7). In Table II by  $K_x$  and  $K_y$  are denoted the Schmidt numbers associated to the isovector pair  $\Gamma_0^+$  and, respectively, to the isoscalar pair  $\Delta_0^+$ . Since in the isovector quartet  $A^+$  all the isovector pairs have the same structure, the like-particle pairs have the Schmidt number  $K_x$ , as the isovector pn pair. By  $K_z$  is denoted the Schmidt number for the odd pair, i.e.,  $\tilde{\Gamma}_0^+$  for T=1 state and  $\tilde{\Delta}_0^+$  for the T=0 state. We recall that the T=0 state is the ground state for <sup>30</sup>P and excited state for <sup>54</sup>Co and <sup>114</sup>La.

From Table II it can be observed that the T=0 pairs are less collective than the isovector T=1 pairs, which is in agreement with the stronger T=1 pairing correlations emerging from the results shown in Table I. In particular, the odd T=0 pair is less collective than the odd

T=1 pair. However, in all nuclei, except  $^{50}\text{Mn}$ , the collectivity of odd T=0 pair is significant and comparable to the collectivity of T=0 pairs in the even-even core of the QCM states. Therefore these calculations indicate that, in general, the T=0 states have not a pure two quasiparticle character.

#### IV. SUMMARY

In this paper we have studied the role of isovector and isoscalar pairing correlations in the lowest T=1 and T=0 states of odd-odd N=Z nuclei. This study is performed in the framework of the QCM approach, which was extended from the even-even to odd-odd nuclei. In the extended QCM formalism the lowest T=0 and T=1 states of odd-odd self-conjugate nuclei are described by a condensate of quartets to which is appended an isoscalar or an isovector proton-neutron pair. As in Ref. [8], the quartets are taken as a linear superposition of an isovector quartet and two collective isoscalar pairs. This model was tested for realistic pairing Hamiltonians and for nuclei with valence nucleons moving above the cores  $^{16}\text{O}$ ,  $^{40}\text{Ca}$  and  $^{100}\text{Sn}$ . The comparison with exact results shows that the energies of the lowest T=1 and T=0 states can be described with high precision by the QCM approach. Taking advantage of the structure of the QCM functions, we have then analyzed the competition between the isovector and isoscalar pairing correlations and the accuracy of various approximations. This analyze indicates that in the nuclei mentioned above the isoscalar pairing correlations are weaker but they coexist together with the isovector correlations in both T=0 and T=1 states. To describe accurately these states is essential to include the isovector pairing through the isovector quartets, in which the isovector pn pairs are coupled together to like-particle pairs. Any approximations in which the contribution of the like-particle pairing is neglected, including the ones in which the T=1 and T=0 states are described by a condensate of isovector pn pairs and, respectively, by a condensate of isoscalar pn pairs, do not describe accurately the pairing correlations in odd-odd N=Z nuclei.

In the present study the lowest T=0 and T=1 states are calculated in the intrinsic system of the axially deformed mean field and therefore they have not a well-defined angular momentum. The restoration of angular momentum will be treated in a future study.

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